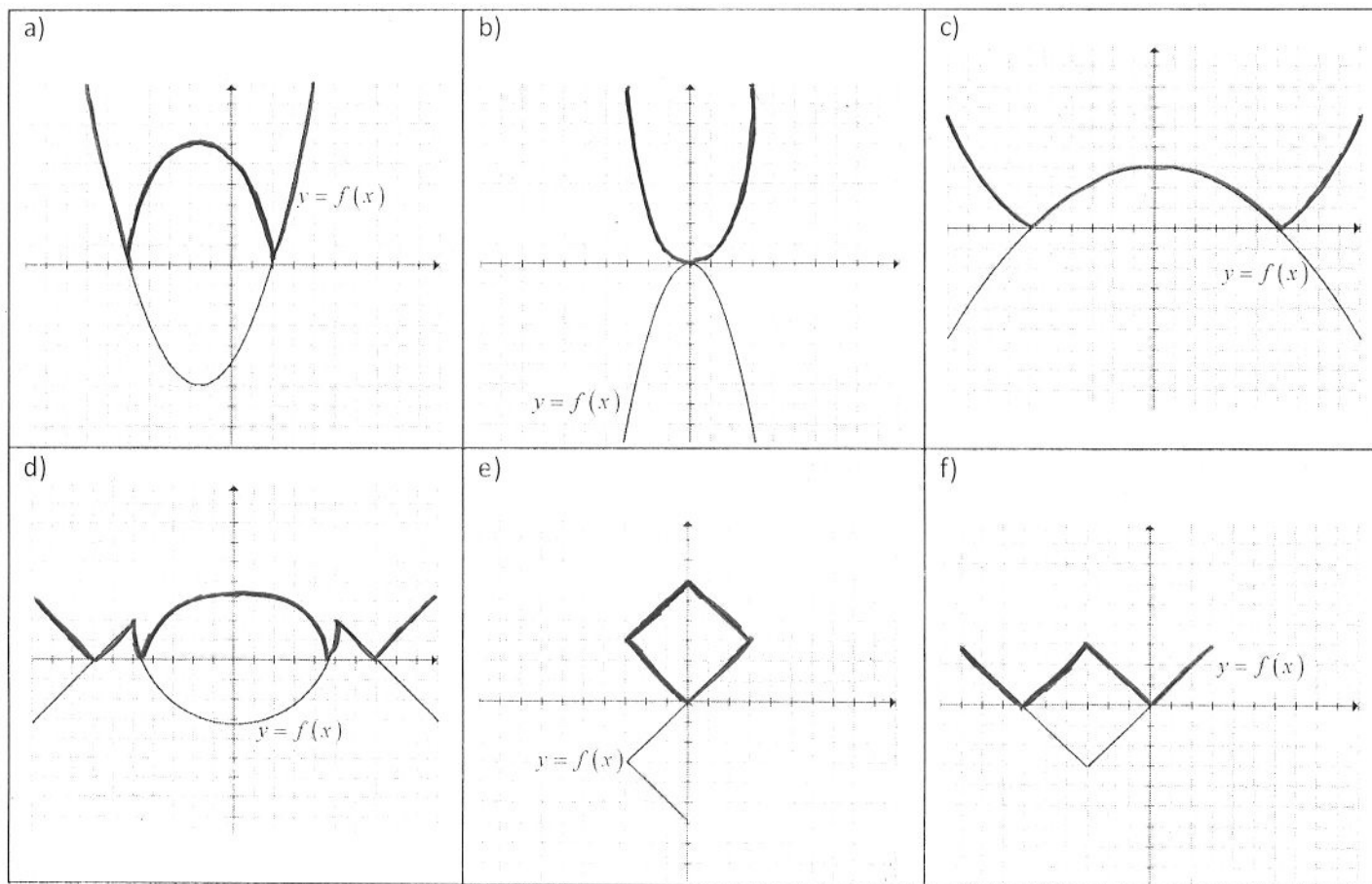


Name: Mahgar Pragesh

Date: Nov 7th 2022

### Section 2.5 Absolute Value of Quadratic Functions and Piece Wise Functions

1. Graph  $y = |f(x)|$  for each function on the same grid:

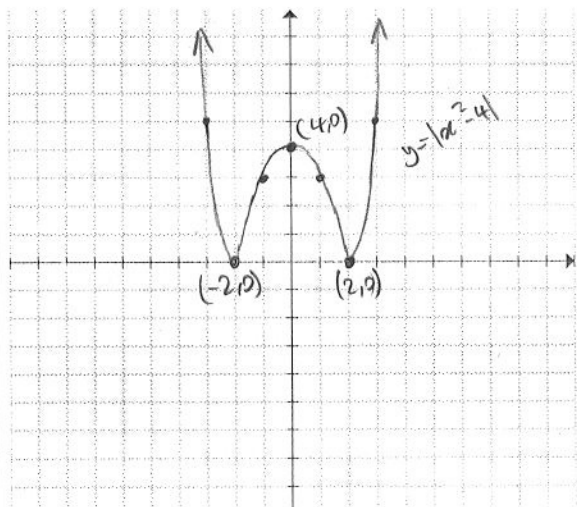


2. Given each equation on the right, indicate which of the graphs on the right is the corresponding one:

|                                    |                                     |         |         |          |
|------------------------------------|-------------------------------------|---------|---------|----------|
| a)<br>$y = - -3x + 7 $<br>(ii)     | b)<br>$y =  (x+3)^2 - 4 $<br>(i, v) | i)<br>  | ii)<br> | iii)<br> |
| c)<br>$y = - (x-3)^2 - 5 $<br>(vi) | d)<br>$y =  3x + 7 $<br>(i)         | iv)<br> | v)<br>  | vi)<br>  |
| e)<br>$y =  (x+3)^2 + 1 $<br>(iii) | f)<br>$y = - -5x - 8  + 4$<br>(v)   |         |         |          |

3. Graph each of the following functions on the grid provided. Get the Domain and Range, state the piece wise function:

a)  $y = |x^2 - 4|$



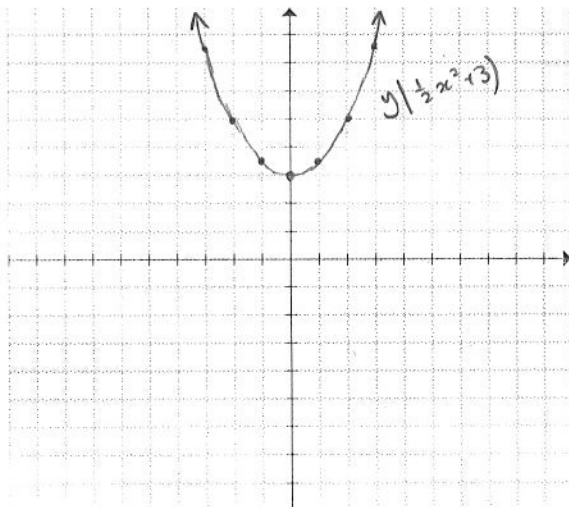
Domain  $x \in \mathbb{R}$

Range:  $y \geq 0$

Piece Wise Function:

$$y = \begin{cases} x^2 - 4 & \text{if } x \leq -2 \text{ or } x \geq 2 \\ -(x^2 - 4) & \text{if } -2 < x < 2 \end{cases}$$

$y = |0.5x^2 + 3|$  inside is always positive in this equation



Domain  $x \in \mathbb{R}$

Range:  $y \geq 3$

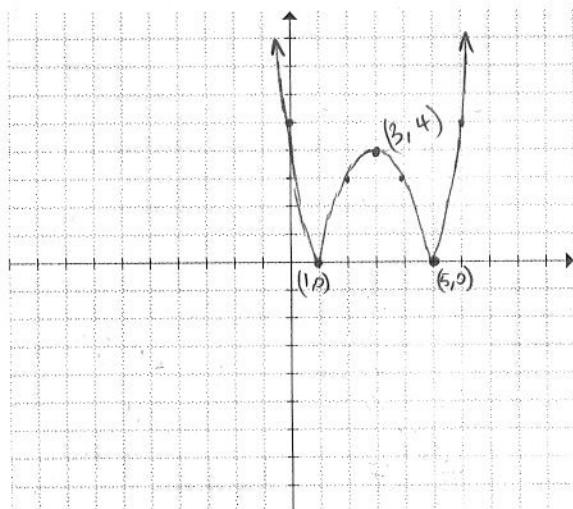
Piece Wise Function:

$$y = \frac{1}{2}x^2 + 3$$

There is no piece wise function.

$y = |(x-3)^2 - 4|$

Roots:  $(x-3)^2 - 4 = 0$   
 $(x-3)^2 = 4$   
 $x-3 = \pm 2$   
 $x_1 = 5$   
 $x_2 = 1$



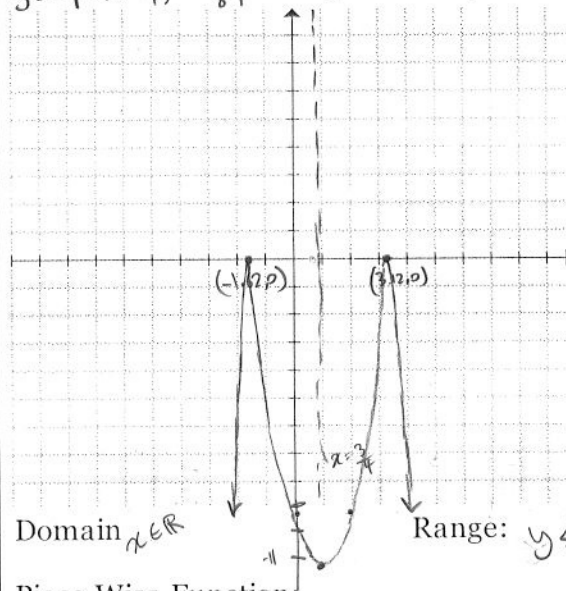
Domain  $x \in \mathbb{R}$

Range:  $y \geq 0$

Piece Wise Function:

$$y = \begin{cases} (x-3)^2 - 4 & \text{if } x \leq 1 \text{ or } x \geq 5 \\ -(x-3)^2 + 4 & \text{if } 1 < x < 5 \end{cases}$$

$y = -|2x^2 - 3x - 10|$  Roots:  $x = \frac{3 \pm \sqrt{9+80}}{4} = \frac{3 \pm \sqrt{89}}{4}$   
 $y = -|2(x - \frac{3}{4})^2 - \frac{89}{8}|$  A.O.S.:  $x = \frac{3}{4}$   $x_1 \approx 3.12$   $x_2 \approx -1.62$



Domain  $x \in \mathbb{R}$

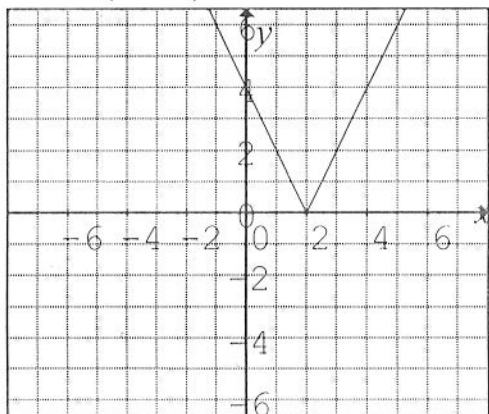
Range:  $y \leq 0$

Piece Wise Function:

$$y = \begin{cases} -2x^2 + 3x - 10 & \text{if } x \leq -1.2 \text{ or } x \geq 3.12 \\ 2x^2 - 3x - 10 & \text{if } -1.2 < x < 3.12 \end{cases}$$

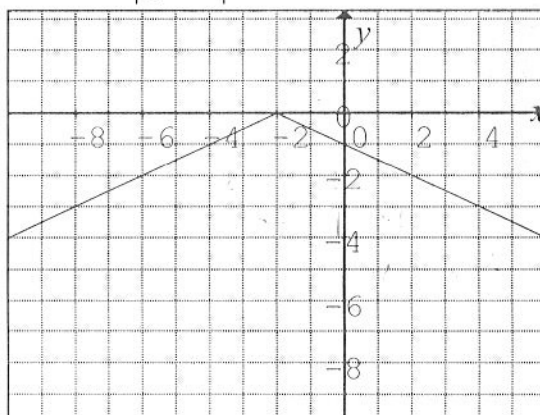
3. Write the piecewise function that represents each absolute value function.

a)  $y = |2x - 4|$



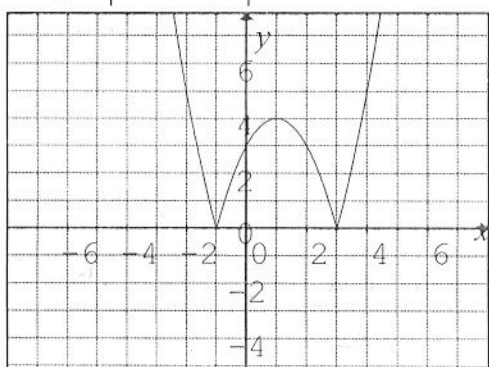
$$y = \begin{cases} 2x - 4 & \text{if } x \geq 2 \\ -2x + 4 & \text{if } x < 2 \end{cases}$$

b)  $y = -\frac{1}{2}|x + 1|$



$$y = \begin{cases} \frac{1}{2}x + 1 & \text{if } x \leq -2 \\ -\frac{1}{2}x - 1 & \text{if } x > -2 \end{cases}$$

c)  $y = |x^2 - 2x - 3|$



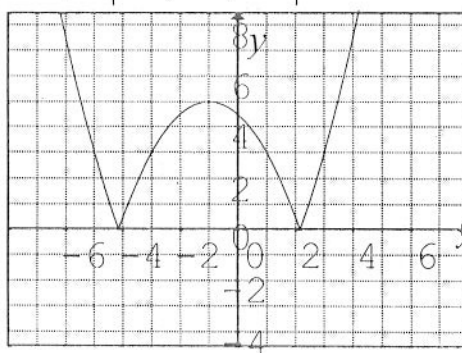
$$y = \begin{cases} x^2 - 2x - 3 & \text{if } x \leq -1 \text{ or } x > 3 \\ -x^2 + 2x + 3 & \text{if } -1 < x \leq 3 \end{cases}$$

d)  $y = |0.5(x+1)^2 - 5|$

Roots:  $\frac{1}{2}(x+1)^2 = 5$

$$x+1 = \pm\sqrt{10}$$

$$x = \pm\sqrt{10} - 1$$



$$y = \begin{cases} \frac{1}{2}(x+1)^2 - 5 & \text{if } x \leq -\sqrt{10} - 1 \text{ or } x > \sqrt{10} - 1 \\ -\frac{1}{2}(x+1)^2 + 5 & \text{if } -\sqrt{10} - 1 < x \leq \sqrt{10} - 1 \end{cases}$$

4. What is the difference between the graphs of  $y = |3x + 1|$  and  $y = -|3x + 1|$ .

One is completely above the x-axis (the first one), and the other is completely below the x-axis. They are mirrored.



5. What is the difference between the graphs of  $y = |3x + 1|$  and  $y = |3x + 1| + 4$ .

The second graph is shifted above the x-axis by 4 units while the first one has a minimum value of 0.



6. The following points (3,5), (-3,-7), (-2,8), (7,-10), and (-3,-9) are on the function  $y = f(x)$ .

What will the coordinates be on the function:  $y = |f(x)|$ ?

(3,5), (-3,7), (-2,8), (7,10), (-3,9)

All of the values of  $y$  must be positive.

$$(x, y) \Rightarrow (x, y)$$

$$(-x, -y) \Rightarrow (-x, y)$$

7. Solve each of the following:

a)  $|x-3| = x-4$

$$x-3 = x-4$$

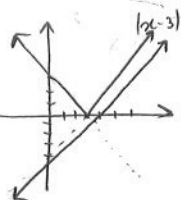
$$-3 = -4$$

No solution

$$x-3 = -(x-4)$$

$$2x = 7$$

$$x = \frac{7}{2} = 3.5$$



$$|3.5-3| = 3.5-4$$

$$|0.5| \neq -0.5$$

Extraneous Solution

b)  $|2x-3| = x+4$

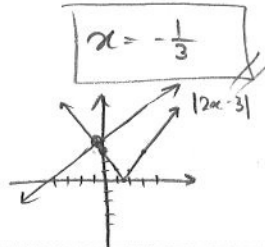
$$2x-3 = -x-4$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$2x-3 = x+4$$

$$x = 7$$



c)  $|x^2+9| = 6x$

$$x^2+9 = 6x$$

$$x^2-6x+9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3$$

$$x^2+9 = -6x$$

$$x^2+6x+9 = 0$$

$$(x+3)^2 = 0$$

$$x = -3$$

$$|9+9| \neq 6(-3) \neq -18$$

Extraneous solution

d)  $|2x^2-x-6| = 2x+1$

$$2x^2-x-6 = 2x+1$$

$$2x^2-3x-7 = 0$$

$$x = \frac{3 \pm \sqrt{9+(4)(2)(7)}}{4} = \frac{3 \pm \sqrt{65}}{4}$$

$$x_1 = \frac{3+\sqrt{65}}{4}$$

$$x_2 = \frac{3-\sqrt{65}}{4}$$

Extraneous solution

$$2x^2-x-6 = -2x-1$$

$$2x^2+x-5 = 0$$

$$x = \frac{-1 \pm \sqrt{1+40}}{4} = \frac{-1 \pm \sqrt{41}}{4}$$

$$x_3 = \frac{-1+\sqrt{41}}{4}$$

$$x_4 = \frac{-1-\sqrt{41}}{4}$$

Extraneous solutions

k)  $|x^2+9| = 6x$

Exact same question as the one above

$$x = 3$$

l)  $|2x^2-x-6| = 2x+1$

Exact same question as above

$$x = \frac{3+\sqrt{65}}{4}$$

$$x = \frac{-1+\sqrt{41}}{4}$$

m)  $12 = |x^2+3|$

$$x^2+3 = 12$$

$$x^2-9 = 0$$

$$x_1 = 3$$

$$x_2 = -3$$

$$x^2+3 = -12$$

$$x^2+15 = 0$$

$$x^2 = -15$$

No real solution here.

$$x = \pm \sqrt{-15}$$

n)  $|x^2-10x| = 24$

$$x^2-10x = 24$$

$$x^2-10x-24 = 0$$

$$(x+2)(x-12) = 0$$

$$x_1 = -2$$

$$x_2 = 12$$

$$x^2-10x = -24$$

$$x^2-10x+24 = 0$$

$$(x-6)(x-4) = 0$$

$$x_1 = 6$$

$$x_2 = 4$$

$$O) |13x - x^2| = 30$$

$$13x - x^2 = 30$$

$$x^2 - 13x + 30 = 0$$

$$(x-10)(x-3) = 0$$

$$x_1 = 10$$

$$x_2 = 3$$

$$13x - x^2 = -30$$

$$x^2 - 13x - 30 = 0$$

$$(x-15)(x+2) = 0$$

$$x_1 = 15$$

$$x_2 = -2$$

$$P) |x^2 - 3x| = 4$$

$$x^2 - 3x = 4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x_1 = 4$$

$$x_2 = -1$$

$$x^2 - 3x = -4$$

$$x^2 - 3x + 4 = 0$$

$$x = \frac{3 \pm \sqrt{9-16}}{2} = \frac{3 \pm \sqrt{-7}}{2} = \emptyset$$

No real roots

8. Find all the value(s) of "x" for which the equation is true:  $|x| = |x+1|$

$$\textcircled{1} \quad x+1 = x$$

$$1 = 0$$

$$\textcircled{2} \quad x+1 = -x$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

(same as  $\textcircled{3}$ )

$$\textcircled{3} \quad -(x+1) = x$$

$$-x-1 = x$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$\textcircled{4} \quad -(x+1) = -x$$

$$x+1 = x$$

$$1 = 0$$

$$x = -\frac{1}{2}$$

$x = -\frac{1}{2}$  is the only value for which  $|x| = |x+1|$  (same as  $\textcircled{3}$ )

9. If  $f(3) = -5$  and  $f(-5) = 7$ , then what is the value of  $|f(-5)| - f^{-1}(-5)$ ?

$$(x, y) \Rightarrow (x, |y|)$$

$$f(x) \quad |f(x)|$$

$$(x, y) = (3, -5)$$

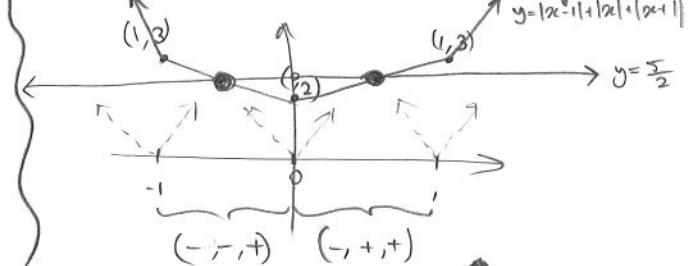
$$(x, y) = (-5, 7)$$

$$|f(-5)| = 7$$

$$f^{-1}(-5) = 3$$

$$7 - 3 = 4$$

Alternate method for #10: Graph it



10. Find the two value(s) that will satisfy the equation:  $|x-1| + |x| + |x+1| = \frac{5}{2}$

Case  $\textcircled{1}$  (+, +, +)

$$x-1+x+x+1 = \frac{5}{2}$$

$$3x = \frac{5}{2}$$

$$x = \frac{5}{6}$$

Extraneous

Case  $\textcircled{2}$  (-, -, -)

$$x-1+x+x+1 = -\frac{5}{2}$$

$$3x = -\frac{5}{2}$$

$$x = -\frac{5}{6}$$

Extraneous

Case  $\textcircled{3}$  (+, +, -)

$$x-1+x-x-1 = \frac{5}{2}$$

$$x-2 = \frac{5}{2}$$

$$x = \frac{9}{2}$$

Extraneous

Case  $\textcircled{4}$  (+, -, +)

$$x-1-x-x+1 = \frac{5}{2}$$

$$x = \frac{5}{2}$$

Extraneous

Case  $\textcircled{5}$  (-, +, +)

$$-x+1+x+x+1 = \frac{5}{2}$$

$$x+2 = \frac{5}{2}$$

$$x = \frac{1}{2}$$

✓

Case  $\textcircled{6}$  (-, -, +)

$$-x-1-x-x+1 = \frac{5}{2}$$

$$-3x = \frac{5}{2}$$

$$x = -\frac{5}{6}$$

✓

Case  $\textcircled{7}$  (-, +, -)

$$-x+1+x-x-1 = \frac{5}{2}$$

$$-x = \frac{5}{2}$$

$$x = -\frac{5}{2}$$

Extraneous

Case  $\textcircled{8}$  (+, -, -)

$$x-1-x-x-1 = \frac{5}{2}$$

$$-2x-2 = \frac{5}{2}$$

$$x = -\frac{9}{4}$$

Extraneous

$$x = \pm \frac{1}{2}$$

all other 6 solutions are extraneous

11. Solve for "x"  $|x^2 - 9x + 20| = |16 - x^2|$

①  $x^2 - 9x + 20 = 16 - x^2$       ②  $x^2 - 9x + 20 = -(16 - x^2)$

$2x^2 - 9x + 4 = 0$

$(2x-1)(x-4) = 0$

Roots:  $x_1 = \frac{1}{2}$   $x_2 = 4$

$x^2 - 9x + 20 = x^2 - 16$

$-9x + 36 = 0$

$9x = 36$

$x = 4$

$x_1 = \frac{1}{2}$   $x_2 = 4$

Note that having  $(-, +) = (+, -)$   
 $(+, +) = (-, -)$  in terms of  
 answers you get.

12. How many ordered pairs of integers (a,b) satisfy this equation?  $|a-2| \times |b-3| = 2$

$|a-2| \times |b-3| = 2$

$1 \times 2 = 2 \Rightarrow a=3 \quad b=5$

$2 \times 1 = 2 \Rightarrow a=4 \quad b=4$

$-1 \times -2 = 2 \Rightarrow a=1 \quad b=1$

$-2 \times -1 = 2 \Rightarrow a=0 \quad b=2$

$|-1| \times 2 = 2 \Rightarrow a=1 \quad b=5$

$|-2| \times 1 = 2 \Rightarrow a=0 \quad b=4$

$2 \times |-1| = 2 \Rightarrow a=4 \quad b=1$

$1 \times |-2| = 2 \Rightarrow a=3 \quad b=2$

8 pairs of possible (a,b)  
 $(a,b) = (3,5), (4,4), (1,1), (0,2), (1,5), (0,4), (4,1), (3,2)$

13. A parabola with equation  $y = ax^2 + bx + c$  is reflected about the x-axis. The parabola and its reflection are translated horizontally five units in opposite directions to become graphs of  $y = f(x)$  and  $y = g(x)$  respectively. Which of the following describes the graph of  $y = (f+g)(x)$ ? i.e.:  $y = f(x) + g(x)$

a) A parabola tangent to the x-axis

b) A parabola not tangent to the x-axis

c) a horizontal line

d) A non-horizontal line

e) the graph of a cubic function

$y = ax^2 + bx + c \Rightarrow y = a(x-p)^2 + q$

$f(x) = a(x-p+5)^2 + q \Rightarrow f(x) = a(x^2 + p^2 + 25 - 2px + 10x - 10p) + q$

$g(x) = -a(x-p-5)^2 - q \Rightarrow g(x) = -a(x^2 + p^2 + 25 - 2xp - 10x + 10p) - q$

$f(x) = ax^2 + ap^2 + 25a - 2px + 10xa - 10pa + q$   
 $g(x) = -ax^2 - ap^2 - 25a + 2xp + 10xa - 10pa - q$  } add them to get  $y = f(x) + g(x)$

$y = f(x) + g(x) = 20xa - 20pa$

Since a is not zero, this is a  
 linear line that is not horizontal! (d)